9. Umocňování komplexních čísel v goniometrickém tvaru, Moivreova věta

př. $z = |z|(\cos\varphi + i\sin\varphi)$

$z^2 = z \cdot z = |z| \cdot |z| [\cos(\varphi+\varphi) + i\sin(\varphi+\varphi)] = |z|^2 (\cos 2\varphi + i \sin 2\varphi)$

VĚTA

Pro $\forall n \in N$ a $\forall z = |z|(\cos\varphi + i\sin\varphi) \in \mathbb{C}$ platí:

$z^n = |z|^n (\cos n\varphi + i\sin n\varphi)$ [abs. hodnotu umocníme, argumenty (úhly) n-krát vynásobíme]

- pokud $|z| = 1$ (z ... komplexní jednotka) $\Rightarrow$ $z^n = \cos n\varphi + i\sin n\varphi$ $\Rightarrow$ MOIVREOVA VĚTA

MOIVREOVA VĚTA [moávrova]

Pro $\forall n \in N$ a lib. reálné číslo φ: $(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi$

- užití: k umocňování kompl. čísel (pro větší $n \in N$ je výpočet mocniny $(a+bi)^n$ pomocí binomické věty zdlouhavý) [BIN. VĚTA - viz KOMBINATORIKA (později)]

- OBĚ VĚTY platí i pro $n \in \mathbb{Z}$

- dodefinujeme: $z^0 = 1 = \cos 0 + i\sin 0$ $\quad z^n = \frac{1}{z^{-n}}$ pro $n \in \mathbb{Z}^-$

př. $(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})^0 = 1$

$(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})^{-2} = \frac{1}{(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})^2} = \frac{\cos 0 + i\sin 0}{\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}} = \cos(-\frac{2\pi}{3}) + i\sin(-\frac{2\pi}{3})$

podle moivreovy věty lze psát rovnou

Příklady

1) Vypočítejte

118/85 a) $(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})^{62} = \cos\frac{62\pi}{3} + i\sin\frac{62\pi}{3} = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = \cos 120° + i\sin 120° =$

[odečteme násobky 2π $\quad \frac{62\pi}{3} = 20\frac{2}{3}\pi = 20\pi + \frac{2\pi}{3}$] $\quad$ II.kv. $\varphi' = 60°$

$= -\cos 60° + i\sin 60° = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

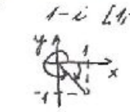
b) $(1-i)^{100} = \{\sqrt{2}[\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})]\}^{100} = (\sqrt{2})^{100} \cdot [\cos(\underbrace{-25\pi}_{+26\pi}) + i\sin(\underbrace{-25\pi}_{+26\pi})] =$ využ. periodič.

$[100 \cdot (-\frac{\pi}{4}) = -25\pi]$

$= 2^{\frac{100}{2}}(\underbrace{\cos\pi}_{-1} + i\underbrace{\sin\pi}_{0}) = 2^{50} \cdot (-1) = -2^{50}$

$\left[\sqrt[m]{a^n} = a^{\frac{n}{m}} \quad (a \geq 0), \quad \sqrt[m]{a} = a^{\frac{1}{m}}\right]$

[$1-i$ [1,-1] na gon. tvar



$\varphi = -\frac{\pi}{4}$ ($\frac{7\pi}{4}$) výhod.

$|z| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$z = \sqrt{2}(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4}))$

($z = \sqrt{2}(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$)]

[NEBO s využ. parity (i period.)

$= (\sqrt{2})^{100}[\cos(-25\pi) + i\sin(-25\pi)] = 2^{50}[\cos(-\pi) + i\sin(-\pi)] =$ využ. paritu

$-25\pi = -24\pi - \pi$

$= 2^{50}(\underbrace{\cos\pi}_{-1} - i\underbrace{\sin\pi}_{0}) = 2^{50}(-1-0) = -2^{50}$]

[NEBO $= (\sqrt{2})^{100}[\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}]^{100} = 2^{50}(\cos\frac{700\pi}{4} + i\sin\frac{700\pi}{4}) = 2^{50}(\cos 175\pi + i\sin 175\pi) =$

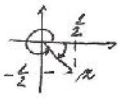
$[175\pi - k \cdot 2\pi = \pi]$ $\quad [\frac{700\pi}{4} = 175\pi]$

$= 2^{50}[\cos\pi + i\sin\pi] = 2^{50}(-1+0) = -2^{50}$

VELKÁ ČÍSLA $\Rightarrow$ pro 4. kv. ASI VÝHODNĚJŠÍ ÚHEL VYJÁDŘIT jako ZÁPORNÝ]

c) $\left(\frac{1}{1+i}\right)^{10}$ nejprve podíl kompl. čísel převedeme na goniom. tvar – lze užanč => umocníme

1. způs. $\frac{1}{1+i} = \frac{1}{1+i}\cdot\frac{1-i}{1-i} = \frac{1-i}{1-i^2} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i \quad [\frac{1}{2}, -\frac{1}{2}]$



$\varphi = -\frac{\pi}{4} \; (\frac{7\pi}{4})$

$|z| = \sqrt{(\frac{1}{2})^2 + (-\frac{1}{2})^2} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2} \; (\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}})$

$= \frac{\sqrt{2}}{2}\left(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})\right) = \frac{\sqrt{2}}{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

nebo $\frac{1}{\sqrt{2}}$ [budeme umocňovat na 10 – výhod.]

2. způs. $\frac{1}{1+i} = \frac{\cos 0 + i\sin 0}{\sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})} = \frac{1}{\sqrt{2}}\left[\cos(\underset{+2\pi}{-\tfrac{\pi}{4}}) + i\sin(\underset{+2\pi}{-\tfrac{\pi}{4}})\right] \Rightarrow \frac{1}{\sqrt{2}}\left[\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right]$

$$\left(\frac{1}{1+i}\right)^{10} = \left\{\frac{1}{\sqrt{2}}\left[\cos(-\tfrac{\pi}{4}) + i\sin(-\tfrac{\pi}{4})\right]\right\}^{10} = \frac{1}{2^{\frac{10}{2}}}\left[\cos(-\tfrac{10\pi}{4}) + i\sin(-\tfrac{10\pi}{4})\right] =$$

$$\rightsquigarrow = \frac{1}{2^5}\left(\cos(\overset{+2\pi}{-\tfrac{\pi}{2}}) + i\sin(\overset{+2\pi}{-\tfrac{\pi}{2}})\right) = \frac{1}{32}\left(\underbrace{\cos\tfrac{3\pi}{2}}_{0} + i\underbrace{\sin\tfrac{3\pi}{2}}_{-1}\right) = \frac{1}{32}(0 - i) = \underline{-\frac{1}{32}i}$$

$$= \frac{1}{2^5}\left(\cos(-\tfrac{\pi}{2}) + i\sin(-\tfrac{\pi}{2})\right) = \frac{1}{32}\left(\underbrace{\cos\tfrac{\pi}{2}}_{0} - i\underbrace{\sin\tfrac{\pi}{2}}_{1}\right) = \frac{1}{32}(0 - i) = \underline{-\frac{1}{32}i}$$

využ. PARITU

POKUD $\left(\frac{\sqrt{2}}{2}\right)^{10} = \frac{(\sqrt{2})^{10}}{2^{10}} = \frac{2^{\frac{10}{2}}}{2^{10}} = \frac{2^5}{2^{10}} = \frac{1}{2^5}$ ZPŮSOBŮ MNOHO – ZÁLEŽÍ NA TOBĚ

$\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)^{10} = \cos\frac{70\pi}{4} + i\sin\frac{70\pi}{4} = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}$ … dále viz výše

$\frac{70\pi}{4} = \frac{35\pi}{2} = 17\frac{1}{2}\pi = 16\pi + \frac{3\pi}{2}$

odečtu sudé násob. 2π

① užitím Moivreovy věty vyjádřete pomocí mocnin $\sin x$, $\cos x$

2.19 a) $\sin 3x$, $\cos 3x$

řeš. komplexní jednotku $(\cos x + i\sin x)$

1. podle Moivreovy věty

$$(\cos x + i\sin x)^3 = \underbrace{\cos 3x}_{Re} + i\underbrace{\sin 3x}_{Im}$$

2. podle binom. věty $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$(\cos x + i\sin x)^3 = \cos^3 x + 3\cos^2 x\cdot\sin x\cdot i + 3\cos x\cdot\underbrace{i^2}_{-1}\sin^2 x + \underbrace{i^3}_{-i}\sin^3 x =$$

$$= \underbrace{\cos^3 x - 3\cos x\sin^2 x}_{Re} + (\underbrace{3\cos^2 x\cdot\sin x - \sin^3 x}_{Im})i$$

3. porovn. Re, Im části

$\sin^2 x + \cos^2 x = 1$
$\sin^2 x = 1 - \cos^2 x$
$\cos^2 x = 1 - \sin^2 x$

$$\cos 3x = \cos^3 x - 3\cos x\cdot\sin^2 x =$$
$$= \cos^3 x - 3\cos x(1 - \cos^2 x) =$$
$$= \cos^3 x - 3\cos x + 3\cos^3 x = \underline{4\cos^3 x - 3\cos x}$$

$$\sin 3x = 3\cos^2 x\cdot\sin x - \sin^3 x =$$
$$= 3(1 - \sin^2 x)\sin x - \sin^3 x$$
$$= 3 - 3\sin^3 x - \sin^3 x = \underline{3 - 4\sin^3 x}$$

b) $\sin 2x$, $\cos 2x$

zú uv. kompl. jednotku $a = \cos x + i \sin x$

1. $a^2 = (\cos x + i \sin x)^2 = \underbrace{\cos 2x}_{Re} + i \underbrace{\sin 2x}_{Im}$ (podle Moiv. věty)

2. $a^2 = \underbrace{(\cos x + i \sin x)^2}_{(a+b)^2} = \cos^2 x + 2 \sin x \cdot \cos x \cdot i + \underbrace{i^2 \sin^2 x}_{-\sin^2 x} =$

$= \underbrace{\cos^2 x - \sin^2 x}_{Re} + \underbrace{2 \sin x \cos x \cdot i}_{Im\ (bez\ i)}$

3. porovnáme Re, Im

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\sin 2x = 2 \sin x \cdot \cos x$$

! GONIOMETRICKÉ VZORCE !

③ Vypočítej

a) $\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)^{50} = \cos \frac{50\pi}{4} + i \sin \frac{50\pi}{4} = \underbrace{\cos \frac{\pi}{2}}_{0} + i \underbrace{\sin \frac{\pi}{2}}_{1} = 0 + 1i = i$

$\left[\frac{50\pi}{4} = \frac{25\pi}{2} = 24\pi + \frac{\pi}{2}\right]$ sudé nás. π nebereme v úvahu

b) $\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)^{31} = \cos \frac{31\pi}{6} + i \sin \frac{31\pi}{6} = \cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} =$

$\left[\frac{31\pi}{6} = 5\pi + \frac{\pi}{6} = 4\pi + \pi + \frac{\pi}{6} \Rightarrow \frac{7\pi}{6}\right]$ sudé nás. π

$= \cos 210° + i \sin 210° = -\underbrace{\cos 30°}_{\frac{\sqrt{3}}{2}} - i \underbrace{\sin 30°}_{\frac{1}{2}} = -\frac{\sqrt{3}}{2} - \frac{1}{2} i$

$\left[\begin{matrix} \ominus \quad \text{III. kv.} \quad \ominus \\ \varphi' = 30° \\ (210° = 180° + \varphi') \end{matrix}\right]$

c) $\left(2\left[\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)\right]\right)^{50} = 2^{50}\left[\cos\left(-\frac{50\pi}{4}\right) + i \sin\left(-\frac{50\pi}{4}\right)\right]$

$= 2^{50}\left[\cos\left(-\frac{\pi}{2}\right)^{+2\pi} + i \sin\left(-\frac{\pi}{2}\right)^{+2\pi}\right] =$ $\left[-\frac{50\pi}{4} = -\frac{25\pi}{2} = -12\pi - \frac{\pi}{2} \Rightarrow -\frac{\pi}{2}\right]$

1 využ. PERIODIC.: $= 2^{50}\left(\underbrace{\cos \frac{3\pi}{2}}_{0} + i \underbrace{\sin \frac{3\pi}{2}}_{-1}\right) = 2^{50}(0 - i) = -2^{50} i$

1 využ. PARITY: $= 2^{50}\left(\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}\right) = 2^{50}(0 - i) = -2^{50} i$

d) $\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)^{70} = \cos \frac{140\pi}{3} + i \sin \frac{140\pi}{3}$ $\left[\frac{140\pi}{3} = 48\frac{2}{3}\pi \Rightarrow \frac{2}{3}\pi\right]$

$= \cos \overset{120°}{\frac{2\pi}{3}} + i \sin \overset{120°}{\frac{2\pi}{3}} = -\underbrace{\cos 60°}_{\frac{1}{2}} + i \underbrace{\sin 60°}_{\frac{\sqrt{3}}{2}} = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$\ominus \quad \varphi = 60° \quad \oplus$ II. kv.

④ vypočítej

zú a) $\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)^{-50} = \cos\left(-\frac{50\pi}{4}\right) + i \sin\left(-\frac{50\pi}{4}\right) =$ $\left[-\frac{50\pi}{4} = -\frac{25\pi}{2} = -12\pi - \frac{\pi}{2} \Rightarrow -\frac{\pi}{2}\right]$

2.18
2.20 $= \cos\left(-\frac{\pi}{2}\right)^{+2\pi} + i \sin\left(-\frac{\pi}{2}\right)^{+2\pi} \rightsquigarrow = \underbrace{\cos \frac{3\pi}{2}}_{0} + i \underbrace{\sin \frac{3\pi}{2}}_{-1} = -i$

$\hookrightarrow \underbrace{\cos \frac{\pi}{2}}_{0} - i \underbrace{\sin \frac{\pi}{2}}_{1} = 0 - i = -i$

b) $\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{-31} = \cos\frac{-31\pi}{6}+i\sin\frac{-31\pi}{6} =$ $\left[-\frac{31\pi}{6} = -5\frac{1}{6}\pi = -4\pi - 1\frac{1}{6}\pi = = -4\pi - \frac{7\pi}{6} \Rightarrow -\frac{7\pi}{6}\right]$

$= \cos\left(-\frac{7\pi}{6}\right)+i\sin\left(-\frac{7\pi}{6}\right) = \cos\frac{7\pi}{6} - i\sin\frac{7\pi}{6} =$

$= \underset{\ominus}{\cos 210^\circ} - i\underset{\ominus}{\sin 210^\circ} = -\cos 30^\circ - i(-\sin 30^\circ) = -\underbrace{\cos 30^\circ}_{\frac{\sqrt{3}}{2}} + \underbrace{\sin 30^\circ}_{\frac{1}{2}} \cdot i = -\frac{\sqrt{3}}{2}+\frac{1}{2}i$

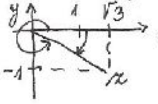
III. kv.
$\varphi' = 30^\circ$

c) $(\sqrt{3}-i)^8 = \left[2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)\right]^8 = 2^8\left[\cos\left(-\frac{8\pi}{6}\right)+i\sin\left(-\frac{8\pi}{6}\right)\right] =$

2.18

$\left[-\frac{8\pi}{6} = -\frac{4\pi}{3}\right]$

[$\sqrt{3}-i$ na gon. tvar $[\sqrt{3}, -1]$



$\varphi \ldots$ IV. kv.

$|z| = \sqrt{(\sqrt{3})^2+(-1)^2} = \sqrt{3+1} = 2$

$\sin\varphi = \frac{b}{|z|}$

$\sin\varphi = \frac{-1}{2}$ $\left[\begin{array}{l}\sin\varphi' = \frac{1}{2} \text{ I. kv.} \\ \varphi' = 30^\circ = \frac{\pi}{6}\end{array}\right]$ φ ve IV. kv. $\varphi = -\frac{\pi}{6}$, $\varphi = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\sqrt{3}-i = 2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$]

$= 2^8\left[\cos\left(\overset{-240^\circ}{-\frac{4\pi}{3}}\right)+i\sin\left(\overset{-240^\circ}{-\frac{4\pi}{3}}\right)\right] = 2^8\left(\cos(-240^\circ)+i\sin(-240^\circ)\right) =$

$\rightsquigarrow$ s využ. PARITY $\cos(-x) = \cos x$, $\sin(-x) = -\sin x$

$= 2^8(\cos 240^\circ - i\sin 240^\circ) = 2^8(-\cos 60^\circ + i\sin 60^\circ) = 2^8\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) = 2^7\cdot 2\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) =$

$\left[\ominus \quad \begin{array}{c}\text{III. kv.} \\ \varphi' = 60^\circ\end{array} \quad \ominus\right]$ $= 2^7(-1+i\sqrt{3}) = 128(-1+i\sqrt{3}) = -128+128\sqrt{3}\,i$

NEBO
$\rightsquigarrow$ s využ. PERIOD.

$= 2^8\left(\cos(\overset{+2\pi\ +360^\circ}{-240^\circ})+i\sin(\overset{+2\pi\ +360^\circ}{-240^\circ})\right) = 2^8(\cos 120^\circ + i\sin 120^\circ) = 2^8(-\underbrace{\cos 60^\circ}_{\frac{1}{2}} + i\underbrace{\sin 60^\circ}_{\frac{\sqrt{3}}{2}})$

$\left[\ominus \quad \begin{array}{c}\text{II. kv.} \\ \varphi' = 60^\circ\end{array} \quad \oplus\right]$

$= 2^8\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) = \ldots$ viz výše

[KDYBY
$\varphi = \frac{11\pi}{6} \Rightarrow 2^8\left[\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right]^8 = 2^8\left(\cos\frac{88\pi}{6}+i\sin\frac{88\pi}{6}\right) =$

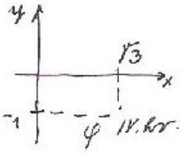
$\left[\frac{88\pi}{6} = 14\frac{4}{6}\pi = 14\frac{2}{3}\pi \Rightarrow \frac{2\pi}{3}\right]$

$= 2^8\left(\cos\overset{120^\circ}{\frac{2\pi}{3}}+i\sin\overset{120^\circ}{\frac{2\pi}{3}}\right) = 2^8(\cos 120^\circ + i\sin 120^\circ) = \ldots$ dále viz výše]

d) $(\sqrt{3}-i)^{-8} = \left[2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)\right]^{-8} = 2^{-8}\left[\cos\left(+\frac{8\pi}{6}\right)+i\sin\frac{8\pi}{6}\right] =$

2.20

[$\sqrt{3}-i$ $[\sqrt{3}, -1]$



IV. kv.

$|z| = \sqrt{(\sqrt{3})^2+(-1)^2}$

$|z| = \sqrt{3+1} = 2$

$\sin\varphi = \frac{b}{|z|} = \frac{-1}{2}$

$\left[\begin{array}{l}\sin\varphi' = \frac{1}{2} \quad \varphi' \ldots \text{I. kv.} \\ \varphi' = 30^\circ = \frac{\pi}{6}\end{array}\right]$

IV. kv. $\varphi = 2\pi - \varphi'$
$\varphi = 2\pi - \frac{\pi}{6}$
$\left(\varphi = -\frac{\pi}{6}\right)$]

$z = 2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$

$= 2^{-8}\left(\cos\underset{240^\circ}{\frac{4\pi}{3}}+i\sin\underset{240}{\frac{4\pi}{3}}\right) = 2^{-8}(\cos 240^\circ + i\sin 240^\circ)$

$\left[\ominus \quad \begin{array}{c}\text{III. kv.} \\ \varphi' = 60^\circ\end{array} \quad \ominus\right]$

$= 2^{-8}(-\cos 60^\circ - i\sin 60^\circ) =$

$= 2^{-8}\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right) = \frac{1}{2^8}\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right) =$

$= \frac{1}{2^8}\cdot\frac{2}{2}\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right) = \frac{1}{2^9}(-1-i\sqrt{3})$

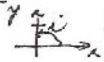
$= 2^{-9}(-1-i\sqrt{3})$

lze i jinak

e) $\left(\frac{1+i}{1-i}\right)^{25} = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{25} = \cos\frac{25\pi}{2} + i\sin\frac{25\pi}{2} = \underbrace{\cos\frac{\pi}{2}}_{0} + i\underbrace{\sin\frac{\pi}{2}}_{1} = i$

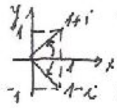
$\left[\frac{25\pi}{2} = 12\pi + \frac{\pi}{2}\right]$

[na gon. tvar

$\frac{1+i}{1-i}\cdot\frac{1+i}{1+i} = \frac{1+2i+\overset{-1}{i^2}}{1-i^2} = \frac{2i}{2} = i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ [ $|i| = 1$, $\varphi = 90° = \frac{\pi}{2}$]

NEBO

$\frac{1+i}{1-i} = \frac{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}{\sqrt{2}\left(\cos(-\frac{\pi}{4}) + i\sin\frac{\pi}{4}\right)} = \cos\left(\frac{\pi}{4} - (-\frac{\pi}{4})\right) + i\sin\left(\frac{\pi}{4} - (-\frac{\pi}{4})\right) = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$



$|1+i| = \sqrt{2}$ $\quad\varphi = \frac{\pi}{4}$

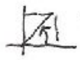
$|1-i| = \sqrt{2}$ $\quad\varphi = -\frac{\pi}{4}$]

f) $\left(\frac{1+i}{1-i}\right)^{-25} = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{-25} = \cos\left(-\frac{25\pi}{2}\right) + i\sin\left(-\frac{25\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right) =$

[na gon. tvar - viz e)] $\rightsquigarrow$ PARITA $= \underbrace{\cos\frac{\pi}{2}}_{0} - i\underbrace{\sin\frac{\pi}{2}}_{1} = -i$

NEBO $\rightsquigarrow$ PERIODA $= \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = 0 - 1i = -i$

g) $(-1+i)^{66} - i(1+i)^{80} = \left[\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right]^{66} - i\left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{80} =$

[$-1+i = [-1, 1]$  $|\alpha| = \sqrt{2}$, $\varphi = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$] [$1+i = [1, 1]$  $|\alpha| = \sqrt{2}$, $\varphi = \frac{\pi}{4}$]

$= (\sqrt{2})^{66}\left[\cos\frac{198\pi}{4} + i\sin\frac{198\pi}{4}\right] - i(\sqrt{2})^{80}\left[\cos\frac{80\pi}{4} + i\sin\frac{80\pi}{4}\right] =$

$\left[\frac{198\pi}{4} = 49\pi + \frac{2\pi}{4} = 48\pi + \pi + \frac{\pi}{2} \Rightarrow \frac{3\pi}{2}\right]$ $\quad\left[\frac{80\pi}{4} = 20\pi = 0 + 20\pi \Rightarrow 0\right]$

$= 2^{\frac{66}{2}}\left(\underbrace{\cos\overset{270°}{\frac{3\pi}{2}}}_{0} + i\underbrace{\sin\frac{3\pi}{2}}_{-1}\right) - i\,2^{\frac{80}{2}}\left(\underbrace{\cos 0}_{1} + i\underbrace{\sin 0}_{0}\right) = 2^{33}(-i) - i\cdot 2^{40} = 2^{33}(-i - 2^{7}i)$

$= 2^{33}i(-1-128) = 2^{33}i(-129) = -129\cdot 2^{33}i$